



## Sesiunea I, iulie 2018

- 1**  $\lim_{x \rightarrow 0} e^{\frac{1}{x}} \cdot \sin x$  is:     A does not exist     B 0     C  $\infty$      D  $-\infty$      E 1

Consider the matrix  $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$  and let  $A^n = \begin{pmatrix} x_n & -y_n \\ y_n & x_n \end{pmatrix}$ ,  $n \in \mathbb{N}^*$ . Denote  $O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$  and  $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ .

- 2**  $2A - A^2$  is:     A  $A + I_2$      B  $I_2$      C  $2I_2$      D  $O_2$      E  $A - I_2$

- 3**  $A^{48}$  is:     A  $O_2$      B  $2^{12}I_2$      C  $2^{48}I_2$      D  $2^{48}A$      E  $2^{24}I_2$

- 4**  $\frac{x_{10}^2 + y_{10}^2}{x_8^2 + y_8^2}$  is:     A 16     B 2     C 8     D 4     E 1

On  $(-1, 1)$  define the operation  $*$  by  $x * y = \frac{2xy + 3(x + y) + 2}{3xy + 2(x + y) + 3}$ ,  $x, y \in (-1, 1)$ .

- 5** The neutral element of  $*$  is:     A 0     B  $\frac{2}{3}$      C  $-\frac{2}{3}$      D  $\frac{1}{3}$      E  $-\frac{1}{3}$

- 6** If the function  $f : (-1, 1) \rightarrow (0, \infty)$ ,  $f(x) = a \frac{1-x}{1+x}$  satisfies  $f(x * y) = f(x)f(y)$ ,  $\forall x, y \in (-1, 1)$ , then  $a$  is:

- A  $-\frac{2}{3}$      B  $\frac{2}{3}$      C  $-\frac{1}{3}$      D  $\frac{1}{5}$      E  $-\frac{1}{5}$

- 7** The number of solutions of the equation  $\underbrace{x * x * \dots * x}_{10 \text{ times } x} = \frac{1}{10}$  is:

- A 2     B 0     C 1     D 10     E 5

- 8** If  $x \in (\pi, 2\pi)$  and  $\cos x = \frac{3}{5}$ , then  $\sin x$  is:

- A  $\frac{3}{4}$      B  $\frac{4}{5}$      C  $-\frac{4}{5}$      D 1     E  $-\frac{3}{4}$

- 9** Consider the sequence with positive terms  $(a_n)_{n \geq 0}$ ,  $a_0 = 1$ ,  $a_1 = a$ ,  $a_{n+1}^3 = a_n^2 a_{n-1}$ ,  $n \geq 1$ . The value of  $a$  for which  $\lim_{n \rightarrow \infty} a_n = 8$  is:

- A 2     B 16     C 8     D 32     E 4

- 10** If  $\lg 5 = a$  and  $\lg 6 = b$ , then  $\log_3 2$  is:

- A  $\frac{1+a}{a+b+1}$      B  $\frac{1+a}{a-b+1}$      C  $\frac{1-a}{a+b+1}$      D  $\frac{1-a}{a+b-1}$      E  $\frac{1-a}{b-1}$

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$$\lim_{x \rightarrow +\infty} \frac{2 \cdot 9^x + 5^x + 4}{9^{x+1} - 5^x + 2^x}$$

is: **A**  $\frac{2}{9}$  **B** 2 **C** 1 **D**  $\frac{1}{9}$  **E**  $+\infty$ **12**

If  $\alpha \in \mathbb{C} \setminus \mathbb{R}$ ,  $\alpha^3 = 1$ , then  $(1 + \alpha)(1 + \alpha^2)(1 + \alpha^3)(1 + \alpha^4)(1 + \alpha^5)(1 + \alpha^6)$  is:

- A**
- 64
- B**
- 0
- C**
- 16
- D**
- 4
- E**
- $8i$

Calculate:

**13**

$$\int_1^5 \frac{dx}{x+3}$$

- A**
- $\ln 2$
- B**
- $\ln 3$
- C**
- $\ln 4$
- D**
- $\ln 5$
- E**
- $\ln 8$

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$$\int_0^1 \frac{dx}{e^x + e^{-x}}$$

- A**
- $\operatorname{arctg} \frac{e}{e+1}$
- B**
- $\operatorname{arctg} e - \frac{\pi}{4}$
- C**
- $\operatorname{arctg} \frac{e}{e^2+1}$
- D**
- $\ln \frac{e}{e+1}$
- E**
- $\ln(2e)$

**15**

$$\int_0^{\frac{\pi}{4}} \frac{\sin(4x)}{\cos^4 x + \sin^4 x} dx$$

- A**
- $\ln 2$
- B**
- $\pi \ln 4$
- C**
- $\pi \ln 8$
- D**
- $\ln \left(\frac{\pi}{4}\right)$
- E**
- $\ln(\pi e)$

**16**

Let  $\{x\}$  be the fractional part of  $x$ . Then  $\lim_{n \rightarrow \infty} n \int_0^\pi \{x\}^n dx$  is:

- A**
- $\frac{\pi}{2}$
- B**
- 4
- C**
- 2
- D**
- $\pi$
- E**
- 3

**17**

Consider the points  $A(2, 3)$  and  $B(4, 5)$ . The perpendicular bisector of the segment  $[AB]$  has the equation:

- A**
- $2x - y = 2$
- B**
- $2x + y = 10$
- C**
- $x + 2y = 11$
- D**
- $-x + y = 1$
- E**
- $x + y = 7$

**18**

If  $x, y \in \mathbb{R}$  satisfy  $2 \lg(x - 2y) = \lg x + \lg y$ , then the set of all values of  $\frac{x}{y}$  is:

- A**
- {4}
- B**
- {1}
- C**
- {1, 4}
- D**
- {1, 2, 4}
- E**
- $\emptyset$

Consider the point  $A(0, -1)$ , the lines  $d_1: x - y + 1 = 0$ ,  $d_2: 2x - y = 0$  and the points  $B \in d_1$ ,  $C \in d_2$ , such that  $d_1$  and  $d_2$  are medians of the triangle  $ABC$ .

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The intersection of the lines  $d_1$  and  $d_2$  has the coordinates:

- A**
- $(-1, 2)$
- B**
- $(2, 3)$
- C**
- $(1, 2)$
- D**
- $(-1, 0)$
- E**
- $\left(-\frac{1}{2}, -1\right)$

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The point  $B$  has the coordinates:

- A**
- $(3, 6)$
- B**
- $(0, 1)$
- C**
- $(1, 2)$
- D**
- $(-1, 0)$
- E**
- $(-2, -1)$



Consider the polynomial  $P = X^{20} + X^{10} + X^5 + 2$ , with roots  $x_1, x_2, x_3, \dots, x_{20}$ . Denote by  $R$  the remainder of the division of  $P$  by  $X^3 + X$ .

- 21**  $P(i)$  is:     A  $2+i$      B  $1+i$      C  $2$      D  $i$      E  $0$

- 22**  $R$  is:     A  $2+X+X^2$      B  $2+X$      C  $2+X-X^2$      D  $X$      E  $1$

- 23**  $\sum_{k=1}^{20} \frac{1}{x_k - x_k^2}$  is:     A  $\frac{15}{2}$      B  $5$      C  $6$      D  $8$      E  $7$

Consider the equation:  $\cos^3 x \cdot \sin x - \sin^3 x \cdot \cos x = m, m \in \mathbb{R}$ .

- 24** The equation has the solution  $x = \frac{\pi}{2}$  for:

- A  $m = \frac{1}{4}$      B  $m = 1$      C  $m = 0$      D  $m = -1$      E  $m = -\frac{1}{4}$

- 25** The equation has solution if and only if  $m$  belongs to the interval:

- A  $[-1, 1]$      B  $[-4, 4]$      C  $\left[-\frac{1}{2}, \frac{1}{2}\right]$      D  $\left[-\frac{1}{4}, \frac{1}{4}\right]$      E  $[-2, 2]$

- 26** The pair  $(a, b) \in \mathbb{R}^2$  such that  $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + \sqrt{x^2 + 2x + 2} - ax - b) = 0$  is:

- A  $\left(2, \frac{3}{2}\right)$      B  $(-2, -1)$      C  $(-2, -2)$      D  $(2, -2)$      E  $\left(-2, -\frac{3}{2}\right)$

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^3 + ax$ , where  $a$  is a real parameter.

- 27**  $f'(0)$  is:     A  $1+a$      B  $a$      C  $1-a$      D  $1$      E  $0$

- 28** The graph of  $f$  is tangent to  $Ox$  axis if:

- A  $a = 2$      B  $a = -1$      C  $a = 1$      D  $a = 0$      E  $a = 3$

- 29** For  $a = -3$  the number of local extremum points of the function  $g(x) = |f(x)|$ ,  $x \in \mathbb{R}$ , is:

- A  $4$      B  $1$      C  $2$      D  $3$      E  $5$

- 30** For  $a = 1$ ,  $(f^{-1})'(2)$  is:     A  $1/2$      B  $1/4$      C  $1/3$      D  $0$      E  $+\infty$



