Sesiunea I, iulie 2018

1 If $\alpha \in \mathbb{C} \setminus \mathbb{R}$, $\alpha^3 = 1$, then $(1+\alpha)(1+\alpha^2)(1+\alpha^3)(1+\alpha^4)(1+\alpha^5)(1+\alpha^6)$ is:

A 64 B 0 C 16 D 4 E $8i$

On $(-1,1)$ define the operation $*$ by $x * y = \dfrac{2xy+3(x+y)+2}{3xy+2(x+y)+3}$, $x, y \in (-1,1)$.

2 The neutral element of $*$ is: A 0 B $\frac{2}{3}$ C $-\frac{2}{3}$ D $\frac{1}{3}$ E $-\frac{1}{3}$

3 If the function $f : (-1,1) \to (0,\infty)$, $f(x) = a\dfrac{1-x}{1+x}$ satisfies $f(x*y) = f(x)f(y)$, $\forall x, y \in (-1,1)$, then a is:

A $-\frac{2}{3}$ B $\frac{2}{3}$ C $-\frac{1}{3}$ D $\frac{1}{5}$ E $-\frac{1}{5}$

4 The number of solutions of the equation $\underbrace{x * x * \ldots * x}_{10 \text{ times } x} = \dfrac{1}{10}$ is:

A 2 B 0 C 1 D 10 E 5

5 The pair $(a,b) \in \mathbb{R}^2$ such that $\lim\limits_{x\to-\infty}(\sqrt{x^2+x+1}+\sqrt{x^2+2x+2}-ax-b) = 0$ is:

A $\left(2, \frac{3}{2}\right)$ B $(-2,-1)$ C $(-2,-2)$ D $(2,-2)$ E $\left(-2, -\frac{3}{2}\right)$

6 Consider the sequence with positive terms $(a_n)_{n\geq 0}$, $a_0 = 1$, $a_1 = a$, $a_{n+1}^3 = a_n^2 a_{n-1}$, $n \geq 1$. The value of a for which $\lim\limits_{n\to\infty} a_n = 8$ is:

A 2 B 16 C 8 D 32 E 4

Consider the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ and let $A^n = \begin{pmatrix} x_n & -y_n \\ y_n & x_n \end{pmatrix}$, $n \in \mathbb{N}^*$. Denote $O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

7 $2A - A^2$ is: A $A + I_2$ B I_2 C $2I_2$ D O_2 E $A - I_2$

8 A^{48} is: A O_2 B $2^{12}I_2$ C $2^{48}I_2$ D $2^{48}A$ E $2^{24}I_2$

9 $\dfrac{x_{10}^2 + y_{10}^2}{x_8^2 + y_8^2}$ is: A 16 B 2 C 8 D 4 E 1

10 If $x, y \in \mathbb{R}$ satisfy $2\lg(x-2y) = \lg x + \lg y$, then the set of all values of $\dfrac{x}{y}$ is:

A $\{4\}$ B $\{1\}$ C $\{1,4\}$ D $\{1,2,4\}$ E $\emptyset$

11 Consider the points $A(2,3)$ and $B(4,5)$. The perpendicular bisector of the segment $[AB]$ has the equation:

A $2x-y=2$ B $2x+y=10$ C $x+2y=11$ D $-x+y=1$ E $x+y=7$

Let $f:\ \mathbb{R}\to\mathbb{R}$, $f(x)=x^3+ax$, where a is a real parameter.

12 $f'(0)$ is: A $1+a$ B a C $1-a$ D 1 E 0

13 The graph of f is tangent to Ox axis if:

A $a=2$ B $a=-1$ C $a=1$ D $a=0$ E $a=3$

14 For $a=-3$ the number of local extremum points of the function $g(x)=|f(x)|$, $x\in\mathbb{R}$, is:

A 4 B 1 C 2 D 3 E 5

15 For $a=1$, $(f^{-1})'(2)$ is: A $1/2$ B $1/4$ C $1/3$ D 0 E $+\infty$

16 If $x\in(\pi,2\pi)$ and $\cos x=\frac{3}{5}$, then $\sin x$ is:

A $\frac{3}{4}$ B $\frac{4}{5}$ C $-\frac{4}{5}$ D 1 E $-\frac{3}{4}$

Consider the point $A(0,-1)$, the lines d_1: $x-y+1=0$, d_2: $2x-y=0$ and the points $B\in d_1$, $C\in d_2$, such that d_1 and d_2 are medians of the triangle ABC.

17 The intersection of the lines d_1 and d_2 has the coordinates:

A $(-1,2)$ B $(2,3)$ C $(1,2)$ D $(-1,0)$ E $\left(-\frac{1}{2},-1\right)$

18 The point B has the coordinates:

A $(3,6)$ B $(0,1)$ C $(1,2)$ D $(-1,0)$ E $(-2,-1)$

19 If $\lg 5=a$ and $\lg 6=b$, then $\log_3 2$ is:

A $\frac{1+a}{a+b+1}$ B $\frac{1+a}{a-b+1}$ C $\frac{1-a}{a+b+1}$ D $\frac{1-a}{a+b-1}$ E $\frac{1-a}{b-1}$

20 $\lim\limits_{x\to 0} e^{\frac{1}{x}}\cdot\sin x$ is: A does not exist B 0 C ∞ D $-\infty$ E 1

Consider the equation: $\cos^3 x \cdot \sin x - \sin^3 x \cdot \cos x = m, m \in \mathbb{R}$.

21 The equation has the solution $x = \frac{\pi}{2}$ for:

A $m = \frac{1}{4}$ B $m = 1$ C $m = 0$ D $m = -1$ E $m = -\frac{1}{4}$

22 The equation has solution if and only if m belongs to the interval:

A $[-1, 1]$ B $[-4, 4]$ C $\left[-\frac{1}{2}, \frac{1}{2}\right]$ D $\left[-\frac{1}{4}, \frac{1}{4}\right]$ E $[-2, 2]$

Consider the polynomial $P = X^{20} + X^{10} + X^5 + 2$, with roots $x_1, x_2, x_3, \ldots, x_{20}$. Denote by R the remainder of the division of P by $X^3 + X$.

23 $P(i)$ is: A $2 + i$ B $1 + i$ C 2 D i E 0

24 R is: A $2 + X + X^2$ B $2 + X$ C $2 + X - X^2$ D X E 1

25 $\sum_{k=1}^{20} \frac{1}{x_k - x_k^2}$ is: A $\frac{15}{2}$ B 5 C 6 D 8 E 7

Calculate:

26 $\int_1^5 \frac{\mathrm{d}x}{x+3}$ A $\ln 2$ B $\ln 3$ C $\ln 4$ D $\ln 5$ E $\ln 8$

27 $\int_0^1 \frac{\mathrm{d}x}{\mathrm{e}^x + \mathrm{e}^{-x}}$

A $\operatorname{arctg} \frac{\mathrm{e}}{\mathrm{e}+1}$ B $\operatorname{arctg} \mathrm{e} - \frac{\pi}{4}$ C $\operatorname{arctg} \frac{\mathrm{e}}{\mathrm{e}^2+1}$ D $\ln \frac{\mathrm{e}}{\mathrm{e}+1}$ E $\ln(2\mathrm{e})$

28 $\int_0^{\frac{\pi}{4}} \frac{\sin(4x)}{\cos^4 x + \sin^4 x} \mathrm{d}x$ A $\ln 2$ B $\pi \ln 4$ C $\pi \ln 8$ D $\ln\left(\frac{\pi}{4}\right)$ E $\ln(\pi \mathrm{e})$

29 Let $\{x\}$ be the fractional part of x. Then $\lim_{n\to\infty} n \int_0^{\pi} \{x\}^n \mathrm{d}x$ is:

A $\frac{\pi}{2}$ B 4 C 2 D π E 3

30 $\lim_{x\to+\infty} \frac{2 \cdot 9^x + 5^x + 4}{9^{x+1} - 5^x + 2^x}$ is: A $\frac{2}{9}$ B 2 C 1 D $\frac{1}{9}$ E $+\infty$