



Sesiunea I, iulie 2018

Calculate:

1 $\int_1^5 \frac{dx}{x+3}$ A $\ln 2$ B $\ln 3$ C $\ln 4$ D $\ln 5$ E $\ln 8$

2 $\int_0^1 \frac{dx}{e^x + e^{-x}}$
 A $\operatorname{arctg} \frac{e}{e+1}$ B $\operatorname{arctg} e - \frac{\pi}{4}$ C $\operatorname{arctg} \frac{e}{e^2+1}$ D $\ln \frac{e}{e+1}$ E $\ln(2e)$

3 $\int_0^{\frac{\pi}{4}} \frac{\sin(4x)}{\cos^4 x + \sin^4 x} dx$ A $\ln 2$ B $\pi \ln 4$ C $\pi \ln 8$ D $\ln\left(\frac{\pi}{4}\right)$ E $\ln(\pi e)$

4 Let $\{x\}$ be the fractional part of x . Then $\lim_{n \rightarrow \infty} n \int_0^\pi \{x\}^n dx$ is:
 A $\frac{\pi}{2}$ B 4 C 2 D π E 3

5 $\lim_{x \rightarrow +\infty} \frac{2 \cdot 9^x + 5^x + 4}{9^{x+1} - 5^x + 2^x}$ is: A $\frac{2}{9}$ B 2 C 1 D $\frac{1}{9}$ E $+\infty$

Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^3 + ax$, where a is a real parameter.

6 $f'(0)$ is: A $1+a$ B a C $1-a$ D 1 E 0

7 The graph of f is tangent to Ox axis if:
 A $a = 2$ B $a = -1$ C $a = 1$ D $a = 0$ E $a = 3$

8 For $a = -3$ the number of local extremum points of the function $g(x) = |f(x)|$, $x \in \mathbb{R}$, is:
 A 4 B 1 C 2 D 3 E 5

9 For $a = 1$, $(f^{-1})'(2)$ is: A $1/2$ B $1/4$ C $1/3$ D 0 E $+\infty$

Consider the point $A(0, -1)$, the lines $d_1: x - y + 1 = 0$, $d_2: 2x - y = 0$ and the points $B \in d_1$, $C \in d_2$, such that d_1 and d_2 are medians of the triangle ABC .

10 The intersection of the lines d_1 and d_2 has the coordinates:
 A $(-1, 2)$ B $(2, 3)$ C $(1, 2)$ D $(-1, 0)$ E $\left(-\frac{1}{2}, -1\right)$

11 The point B has the coordinates:
 A $(3, 6)$ B $(0, 1)$ C $(1, 2)$ D $(-1, 0)$ E $(-2, -1)$

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Consider the points $A(2, 3)$ and $B(4, 5)$. The perpendicular bisector of the segment $[AB]$ has the equation:

- A $2x - y = 2$ B $2x + y = 10$ C $x + 2y = 11$ D $-x + y = 1$ E $x + y = 7$

Consider the polynomial $P = X^{20} + X^{10} + X^5 + 2$, with roots $x_1, x_2, x_3, \dots, x_{20}$. Denote by R the remainder of the division of P by $X^3 + X$.

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$P(i)$ is: A $2+i$ B $1+i$ C 2 D i E 0

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R is: A $2+X+X^2$ B $2+X$ C $2+X-X^2$ D X E 1

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$\sum_{k=1}^{20} \frac{1}{x_k - x_k^2}$ is: A $\frac{15}{2}$ B 5 C 6 D 8 E 7

Consider the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ and let $A^n = \begin{pmatrix} x_n & -y_n \\ y_n & x_n \end{pmatrix}$, $n \in \mathbb{N}^*$. Denote $O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

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$2A - A^2$ is: A $A + I_2$ B I_2 C $2I_2$ D O_2 E $A - I_2$

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A^{48} is: A O_2 B $2^{12}I_2$ C $2^{48}I_2$ D $2^{48}A$ E $2^{24}I_2$

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$\frac{x_{10}^2 + y_{10}^2}{x_8^2 + y_8^2}$ is: A 16 B 2 C 8 D 4 E 1

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The pair $(a, b) \in \mathbb{R}^2$ such that $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + \sqrt{x^2 + 2x + 2} - ax - b) = 0$ is:

- A $\left(2, \frac{3}{2}\right)$ B $(-2, -1)$ C $(-2, -2)$ D $(2, -2)$ E $\left(-2, -\frac{3}{2}\right)$

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$\lim_{x \rightarrow 0} e^{\frac{1}{x}} \cdot \sin x$ is: A does not exist B 0 C ∞ D $-\infty$ E 1

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Consider the sequence with positive terms $(a_n)_{n \geq 0}$, $a_0 = 1$, $a_1 = a$, $a_{n+1}^3 = a_n^2 a_{n-1}$, $n \geq 1$. The value of a for which $\lim_{n \rightarrow \infty} a_n = 8$ is:

- A 2 B 16 C 8 D 32 E 4



Consider the equation: $\cos^3 x \cdot \sin x - \sin^3 x \cdot \cos x = m, m \in \mathbb{R}$.

- 22** The equation has the solution $x = \frac{\pi}{2}$ for:

- (A) $m = \frac{1}{4}$ (B) $m = 1$ (C) $m = 0$ (D) $m = -1$ (E) $m = -\frac{1}{4}$

- 23** The equation has solution if and only if m belongs to the interval:

- (A) $[-1, 1]$ (B) $[-4, 4]$ (C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (E) $[-2, 2]$

- 24** If $x \in (\pi, 2\pi)$ and $\cos x = \frac{3}{5}$, then $\sin x$ is:

- (A) $\frac{3}{4}$ (B) $\frac{4}{5}$ (C) $-\frac{4}{5}$ (D) 1 (E) $-\frac{3}{4}$

- 25** If $\lg 5 = a$ and $\lg 6 = b$, then $\log_3 2$ is:

- (A) $\frac{1+a}{a+b+1}$ (B) $\frac{1+a}{a-b+1}$ (C) $\frac{1-a}{a+b+1}$ (D) $\frac{1-a}{a+b-1}$ (E) $\frac{1-a}{b-1}$

- 26** If $x, y \in \mathbb{R}$ satisfy $2\lg(x-2y) = \lg x + \lg y$, then the set of all values of $\frac{x}{y}$ is:

- (A) {4} (B) {1} (C) {1, 4} (D) {1, 2, 4} (E) \emptyset

- 27** If $\alpha \in \mathbb{C} \setminus \mathbb{R}$, $\alpha^3 = 1$, then $(1+\alpha)(1+\alpha^2)(1+\alpha^3)(1+\alpha^4)(1+\alpha^5)(1+\alpha^6)$ is:

- (A) 64 (B) 0 (C) 16 (D) 4 (E) $8i$

On $(-1, 1)$ define the operation $*$ by $x * y = \frac{2xy + 3(x+y) + 2}{3xy + 2(x+y) + 3}, x, y \in (-1, 1)$.

- 28** The neutral element of $*$ is: (A) 0 (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $\frac{1}{3}$ (E) $-\frac{1}{3}$

- 29** If the function $f : (-1, 1) \rightarrow (0, \infty)$, $f(x) = a \frac{1-x}{1+x}$ satisfies $f(x * y) = f(x)f(y)$, $\forall x, y \in (-1, 1)$, then a is:

- (A) $-\frac{2}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{5}$ (E) $-\frac{1}{5}$

- 30** The number of solutions of the equation $\underbrace{x * x * \dots * x}_{10 \text{ times } x} = \frac{1}{10}$ is:

- (A) 2 (B) 0 (C) 1 (D) 10 (E) 5

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